

(証明viii)

仮定より

$$OM_1 = 2OX_1 \quad \cdots(d.8.1)$$

$$OM_2 = 2OX_2 \quad \cdots(d.8.2)$$

$$OM_3 = 2OX_3 \quad \cdots(d.8.3)$$

$$OM_4 = 2OX_4 \quad \cdots(d.8.4)$$

(d.7.59)(d.8.1)(d.8.3)より

$$2OX_3 = (2OX_1)^2 - 2 \quad \cdots(d.8.5)$$

(d.8.5)より

$$OX_3 = 2OX_1^2 - 1 \quad \cdots(d.8.6)$$

(d.7.32)(d.8.2)(d.8.3)より

$$2OX_2 = (2OX_3)^2 - 2 \quad \cdots(d.8.7)$$

(d.8.7)より

$$OX_2 = 2OX_3^2 - 1 \quad \cdots(d.8.8)$$

(d.7.62)(d.8.2)(d.8.4)より

$$2OX_4 = 2 - (2OX_2)^2 \quad \cdots(d.8.9)$$

(d.8.9)より

$$OX_4 = 1 - 2OX_2^2 \quad \cdots(d.8.10)$$

(d.7.29)(d.8.1)(d.8.4)より

$$2OX_1 = (2OX_4)^2 - 2 \quad \cdots(d.8.11)$$

(d.8.11)より

$$OX_1 = 2OX_4^2 - 1 \quad \cdots(d.8.12)$$

[命題 1] と(d.8.6)より

$$\angle Z_1 OY_5 = 2\angle Z_1 OY_1 \quad \cdots(d.8.13)$$

[命題 1] と(d.8.8)より

$$\angle Z_1 OY_3 = 2\angle Z_1 OY_5 \quad \cdots(d.8.14)$$

[命題 2] と(d.8.10)より

$$\angle Z_1 OY_7 = 2\angle Z_1 OY_3 \quad \cdots(d.8.15)$$

[命題 4] と(d.8.12)より

$$\text{優角 } Z_1 OY_2 = 2\angle Z_1 OY_7 \quad \cdots(d.8.16)$$

(d.8.13)(d.8.14)より

$$\angle Z_1 OY_3 = 4\angle Z_1 OY_1 \quad \cdots (d.8.17)$$

(d.8.15)(d.8.17)より

$$\angle Z_1 OY_7 = 8\angle Z_1 OY_1 \quad \cdots (d.8.18)$$

(d.8.16)(d.8.18)より

$$\text{優角 } \angle Z_1 OY_2 = 16\angle Z_1 OY_1 \quad \cdots (d.8.19)$$

対称性（あるいは $\triangle Z_1 OY_1$ と $\triangle Z_1 OY_2$ とが合同であること）より

$$\angle Z_1 OY_2 = \angle Z_1 OY_1 \quad \cdots (d.8.20)$$

同じ角の優角と劣角だから

$$\text{優角 } \angle Z_1 OY_2 + \angle Z_1 OY_2 = 360^\circ \quad \cdots (d.8.21)$$

(d.8.19)(d.8.20)(d.8.21)より

$$16\angle Z_1 OY_1 + \angle Z_1 OY_1 = 360^\circ \quad \cdots (d.8.22)$$

(d.8.22)より

$$17\angle Z_1 OY_1 = 360^\circ \quad \cdots (d.8.23)$$

(d.8.23)より

$$\angle Z_1 OY_1 = \left( \frac{360}{17} \right)^\circ \quad \cdots (d.8.24)$$

(証明おわり)