

(証明 vii)

(d.5.35)より、次の関係式を満たす点 M_1' をとることができる。

$$OM_1' = OM_4^2 - 2 \quad \cdots(d.7.1)$$

(d.5.31)より、次の関係式を満たす点 M_2' をとることができる。

$$OM_2' = OM_3^2 - 2 \quad \cdots(d.7.2)$$

(d.7.1)(d.7.2)より

$$OM_1' + OM_2' = (OM_4^2 - 2) + (OM_3^2 - 2) \quad \cdots(d.7.3)$$

(d.7.3)より

$$OM_1' + OM_2' = OM_3^2 + OM_4^2 - 4 \quad \cdots(d.7.4)$$

(d.3.37)(d.3.40)(d.7.4)より

$$OM_1' + OM_2' = (OI_3 - OI_2 \cdot OM_3) + (OI_2 \cdot OM_4 + OI_3) - 4 \quad \cdots(d.7.5)$$

(d.7.5)より

$$OM_1' + OM_2' = OI_2 (OM_4 - OM_3) + 2OI_3 - 4 \quad \cdots(d.7.6)$$

(d.3.22)(d.7.6)より

$$OM_1' + OM_2' = OI_2 \cdot OI_2 + 2OI_3 - 4 \quad \cdots(d.7.7)$$

(d.7.7)より

$$OM_1' + OM_2' = OI_2^2 + 2OI_3 - 4 \quad \cdots(d.7.8)$$

(d.6.58)(d.7.8)より

$$OM_1' + OM_2' = OI_1 \quad \cdots(d.7.9)$$

(d.7.1)(d.7.2)より

$$OM_1' \cdot OM_2' = (OM_4^2 - 2)(OM_3^2 - 2) \quad \cdots(d.7.10)$$

(d.7.10)より

$$OM_1' \cdot OM_2' = (OM_3 \cdot OM_4)^2 - 2OM_3^2 - 2OM_4^2 + 4 \quad \cdots(d.7.11)$$

(d.3.24)(d.7.11)より

$$OM_1' \cdot OM_2' = OI_3^2 - 2OM_3^2 - 2OM_4^2 + 4 \quad \cdots(d.7.12)$$

(d.3.37)(d.3.40)(d.7.12)より

$$OM_1' \cdot OM_2' = OI_3^2 - 2(OI_3 - OI_2 \cdot OM_3) - 2(OI_2 \cdot OM_4 + OI_3) + 4 \quad \cdots(d.7.13)$$

(d.7.13)より

$$OM_1' \cdot OM_2' = OI_3^2 - 2OI_2(OM_4 - OM_3) - 4OI_3 + 4 \quad \cdots(d.7.14)$$

(d.3.22)(d.7.14)より

$$OM_1' \cdot OM_2' = OI_3^2 - 2OI_2 \cdot OI_2 - 4OI_3 + 4 \quad \cdots(d.7.15)$$

(d.7.15)より

$$OM_1' \cdot OM_2' = OI_3^2 - 2(OI_2^2 + 2OI_3 - 4) - 4 \quad \cdots(d.7.16)$$

(d.6.58)(d.7.16)より

$$OM_1' \cdot OM_2' = OI_3^2 - 2OI_1 - 4 \quad \cdots(d.7.17)$$

(d.6.82)(d.7.17)より

$$OM_1' \cdot OM_2' = OI_4 \quad \cdots(d.7.18)$$

(d.3.8)(d.7.9)より

$$OM_1' + OM_2' = OM_1 + OM_2 \quad \cdots(d.7.19)$$

(d.3.10)(d.7.18)より

$$OM_1' \cdot OM_2' = OM_1 \cdot OM_2 \quad \cdots(d.7.20)$$

(d.5.36)(d.7.2)より

$$OM_2' < OM_1 \quad \cdots(d.7.21)$$

(d.5.37)(d.7.1)より

$$OM_2 < OM_1' \quad \cdots(d.7.22)$$

計算により

$$(OM_1 - OM_1') (OM_1 - OM_2') = OM_1^2 - (OM_1' + OM_2') OM_1 + OM_1' \cdot OM_2' \quad \cdots(d.7.23)$$

(d.7.19)(d.7.20)(d.7.23)より

$$(OM_1 - OM_1') (OM_1 - OM_2') = OM_1^2 - (OM_1 + OM_2) OM_1 + OM_1 \cdot OM_2 \quad \cdots(d.7.24)$$

(d.7.24)より

$$\left(OM_1 - OM_1'\right)\left(OM_1 - OM_2'\right) = 0 \quad \cdots(d.7.25)$$

(d.7.21)より

$$OM_1 - OM_2' > 0 \quad \cdots(d.7.26)$$

(d.7.25)(d.7.26)より

$$OM_1 - OM_1' = 0 \quad \cdots(d.7.27)$$

(d.7.27)より

$$OM_1 = OM_1' \quad \cdots(d.7.28)$$

(d.7.1)(d.7.28)より

$$OM_1 = OM_4'^2 - 2 \quad \cdots(d.7.29)$$

(d.7.19)(d.7.28)より

$$OM_1 + OM_2' = OM_1 + OM_2 \quad \cdots(d.7.30)$$

(d.7.30)より

$$OM_2 = OM_2' \quad \cdots(d.7.31)$$

(d.7.31)(d.7.2)より

$$OM_2 = OM_3'^2 - 2 \quad \cdots(d.7.32)$$

(d.5.23)より、次の関係式を満たす点 M_3' をとることができる。

$$OM_3' = OM_1'^2 - 2 \quad \cdots(d.7.33)$$

(d.5.27)より、次の関係式を満たす点 M_4' をとることができる。

$$OM_4' = 2 - OM_2'^2 \quad \cdots(d.7.34)$$

(d.7.33)(d.7.34)より

$$OM_4' - OM_3' = (2 - OM_2'^2) - (OM_1'^2 - 2) \quad \cdots(d.7.35)$$

(d.7.35)より

$$OM_4' - OM_3' = -OM_1'^2 - OM_2'^2 + 4 \quad \cdots(d.7.36)$$

(d.3.31)(d.7.34)(d.7.36)より

$$OM_4' - OM_3' = -(OI_1 \cdot OM_1 - OI_4) - (OI_1 \cdot OM_2 - OI_4) + 4 \quad \cdots(d.7.37)$$

(d.7.37)より

$$OM_4' - OM_3' = -OI_1(OM_1 + OM_2) + 2OI_4 + 4 \quad \cdots(d.7.38)$$

(d.3.8)(d.7.38)より

$$OM_4' - OM_3' = -OI_1 \cdot OI_1 + 2OI_4 + 4 \quad \cdots(d.7.39)$$

(d.7.39)より

$$OM_4' - OM_3' = -OI_1^2 + 2OI_4 + 4 \quad \cdots(d.7.40)$$

(d.6.66)(d.7.40)より

$$OM_4' - OM_3' = OI_2 \quad \cdots(d.7.41)$$

(d.7.33)(d.7.34)より

$$OM_3' \cdot OM_4' = (OM_1^2 - 2)(2 - OM_2^2) \quad \cdots(d.7.42)$$

(d.7.42)より

$$OM_3' \cdot OM_4' = -(OM_1 \cdot OM_2)^2 + 2OM_1^2 + 2OM_2^2 - 4 \quad \cdots(d.7.43)$$

(d.3.10)(d.7.43)より

$$OM_3' \cdot OM_4' = -OI_4^2 + 2OM_1^2 + 2OM_2^2 - 4 \quad \cdots(d.7.44)$$

(d.3.31)(d.3.34)(d.7.44)より

$$OM_3' \cdot OM_4' = -OI_4^2 + 2(OI_1 \cdot OM_1 - OI_4) + 2(OI_1 \cdot OM_2 - OI_4) - 4 \quad \cdots(d.7.45)$$

(d.7.45)より

$$OM_3' \cdot OM_4' = -OI_4^2 + 2OI_1(OM_1 + OM_2) - 4OI_4 - 4 \quad \cdots(d.7.46)$$

(d.3.8)(d.7.46)より

$$OM_3' \cdot OM_4' = -OI_4^2 + 2OI_1 \cdot OI_1 - 4OI_4 - 4 \quad \cdots(d.7.47)$$

(d.7.47)より

$$OM_3' \cdot OM_4' = -OI_4^2 - 2(-OI_1^2 + 2OI_4 + 4) + 4 \quad \cdots(d.7.48)$$

(d.6.66)(d.7.48)より

$$OM_3' \cdot OM_4' = -OI_4^2 - 2OI_2 + 4 \quad \cdots(d.7.49)$$

(d.6.74)(d.7.49)より

$$OM_3' \cdot OM_4' = OI_3 \quad \cdots(d.7.50)$$

(d.3.22)(d.7.41)より

$$OM_4' - OM_3' = OM_4 - OM_3 \quad \cdots(d.7.51)$$

(d.3.24)(d.7.50)より

$$OM_3' \cdot OM_4' = OM_2 \cdot OM_4 \quad \cdots(d.7.52)$$

計算により

$$\begin{aligned} (OM_3 - OM_3') (OM_3 + OM_4') &= OM_3^2 + (OM_4' - OM_3') OM_3 - OM_3' \cdot OM_4' \\ &\cdots(d.7.53) \end{aligned}$$

(d.7.51)(d.7.52)(d.7.53)より

$$\begin{aligned} (OM_3 - OM_3') (OM_3 + OM_4') &= OM_3^2 + (OM_4 - OM_3) OM_3 - OM_3 \cdot OM_4 \\ &\cdots(d.7.54) \end{aligned}$$

(d.7.54)より

$$(OM_3 - OM_3') (OM_3 + OM_4') = 0 \quad \cdots(d.7.55)$$

$OM_3 > 0$ 、 $OM_4' > 0$ より

$$OM_3 + OM_4' > 0 \quad \cdots(d.7.56)$$

(d.7.55)(d.7.56)より

$$OM_3 - OM_3' = 0 \quad \cdots(d.7.57)$$

(d.7.57)より

$$OM_3 = OM_3' \quad \cdots(d.7.58)$$

(d.7.33)(d.7.58)より

$$OM_3 = OM_1^2 - 2 \quad \cdots(d.7.59)$$

(d.7.51)(d.7.58)より

$$OM_4' - OM_3 = OM_4 - OM_3 \quad \cdots(d.7.60)$$

(d.7.60)より

$$OM_4 = OM_4' \quad \cdots(d.7.61)$$

(d.7.34)(d.7.61)より

$$OM_4 = 2 - OM_2^2 \quad \dots(d.7.62)$$