（証明vi）

(d.5.6)より、次の関係式を満たす点$\mathrm{I}_1{}'$をとることができる。

$$\mathrm{OI}_1{}' = \frac{1}{2}\left(\mathrm{OE}_1 \cdot \mathrm{OI}_3 + \mathrm{OE}_1 - 2\right) \quad \cdots(\mathrm{d.6.1})$$

(d.5.12)より、次の関係式を満たす点$\mathrm{I}_2{}'$をとることができる。

$$\mathrm{OI}_2{}' = \frac{1}{2}\left(\mathrm{OE}_1 \cdot \mathrm{OI}_4 - \mathrm{OE}_1 + 2\right) \quad \cdots(\mathrm{d.6.2})$$

(d.6.1)(d.6.2)より

$$\mathrm{OI}_1{}' - \mathrm{OI}_2{}' = \frac{1}{2}\left(\mathrm{OE}_1 \cdot \mathrm{OI}_3 + \mathrm{OE}_1 - 2\right) - \frac{1}{2}\left(\mathrm{OE}_1 \cdot \mathrm{OI}_4 - \mathrm{OE}_1 + 2\right) \quad \cdots(\mathrm{d.6.3})$$

(d.6.3)より

$$\mathrm{OI}_1{}' - \mathrm{OI}_2{}' = \frac{1}{2}\mathrm{OE}_1\left(\mathrm{OI}_3 - \mathrm{OI}_4\right) + \mathrm{OE}_1 - 2 \quad \cdots(\mathrm{d.6.4})$$

(d.2.20)(d.6.4)より

$$\mathrm{OI}_1{}' - \mathrm{OI}_2{}' = \frac{1}{2}\mathrm{OE}_1 \cdot \mathrm{OE}_2 + \mathrm{OE}_1 - 2 \quad \cdots(\mathrm{d.6.5})$$

(d.1.14)(d.6.5)より

$$\mathrm{OI}_1{}' - \mathrm{OI}_2{}' = \frac{1}{2} \cdot 4 + \mathrm{OE}_1 - 2 \quad \cdots(\mathrm{d.6.6})$$

(d.6.6)より

$$\mathrm{OI}_1{}' - \mathrm{OI}_2{}' = \mathrm{OE}_1 \quad \cdots(\mathrm{d.6.7})$$

(d.6.1)(d.6.2)より

$$\mathrm{OI}_1{}' \cdot \mathrm{OI}_2{}' = \frac{1}{2}\left(\mathrm{OE}_1 \cdot \mathrm{OI}_3 + \mathrm{OE}_1 - 2\right) \cdot \frac{1}{2}\left(\mathrm{OE}_1 \cdot \mathrm{OI}_4 - \mathrm{OE}_1 + 2\right) \quad \cdots(\mathrm{d.6.8})$$

(d.6.8)より

$$\mathrm{OI}_1{}' \cdot \mathrm{OI}_2{}' = \frac{1}{4}\mathrm{OE}_1{}^2 \cdot \mathrm{OI}_3 \cdot \mathrm{OI}_4 + \frac{1}{4}\mathrm{OE}_1\left(-\mathrm{OE}_1 + 2\right)\left(\mathrm{OI}_3 - \mathrm{OI}_4\right) - \frac{1}{4}\left(-\mathrm{OE}_1 + 2\right)^2$$

$\cdots(\mathrm{d.6.9})$

(d.2.20)(d.2.22)(d.6.9)より

$$\mathrm{OI}_1{}' \cdot \mathrm{OI}_2{}' = \frac{1}{4}\mathrm{OE}_1{}^2 \cdot 1 + \frac{1}{4}\mathrm{OE}_1\left(-\mathrm{OE}_1 + 2\right)\mathrm{OE}_2 - \frac{1}{4}\left(-\mathrm{OE}_1 + 2\right)^2 \quad \cdots(\mathrm{d.6.10})$$

(d.6.10)より

$$\mathrm{OI}_1{}' \cdot \mathrm{OI}_2{}' = \frac{1}{4}\mathrm{OE}_1 \cdot \mathrm{OE}_2\left(-\mathrm{OE}_1 + 2\right) + \mathrm{OE}_1 - 1 \quad \cdots(\mathrm{d.6.11})$$

(d.1.14)(d.6.11)より

$$\mathrm{OI_1}' \cdot \mathrm{OI_2}' = \frac{1}{4} \cdot 4(-\mathrm{OE_1} + 2) + \mathrm{OE_1} - 1 \quad \cdots(\mathrm{d.6.12})$$

(d.6.12)より

$$\mathrm{OI_1}' \cdot \mathrm{OI_2}' = 1 \quad \cdots(\mathrm{d.6.13})$$

(d.2.7)(d.6.7)より

$$\mathrm{OI_1}' - \mathrm{OI_2}' = \mathrm{OI_1} - \mathrm{OI_2} \quad \cdots(\mathrm{d.6.14})$$

(d.2.9)(d.6.13)より

$$\mathrm{OI_1}' \cdot \mathrm{OI_2}' = \mathrm{OI_1} \cdot \mathrm{OI_2} \quad \cdots(\mathrm{d.6.15})$$

計算により

$$\left(\mathrm{OI_1} - \mathrm{OI_1}'\right)\left(\mathrm{OI_1} + \mathrm{OI_2}'\right) = \mathrm{OI_1}^2 - \left(\mathrm{OI_1}' - \mathrm{OI_2}'\right)\mathrm{OI_1} - \mathrm{OI_1}' \cdot \mathrm{OI_2}' \quad \cdots(\mathrm{d.6.16})$$

(d.6.14)(d.6.15)(d.6.16)より

$$\left(\mathrm{OI_1} - \mathrm{OI_1}'\right)\left(\mathrm{OI_1} + \mathrm{OI_2}'\right) = \mathrm{OI_1}^2 - (\mathrm{OI_1} - \mathrm{OI_2})\mathrm{OI_1} - \mathrm{OI_1} \cdot \mathrm{OI_2} \quad \cdots(\mathrm{d.6.17})$$

(d.6.17)より

$$\left(\mathrm{OI_1} - \mathrm{OI_1}'\right)\left(\mathrm{OI_1} + \mathrm{OI_2}'\right) = 0 \quad \cdots(\mathrm{d.6.18})$$

$\mathrm{OI_1} > 0$、$\mathrm{OI_2}' > 0$より

$$\mathrm{OI_1} + \mathrm{OI_2}' > 0 \quad \cdots(\mathrm{d.6.19})$$

(d.6.18)(d.6.19)より

$$\mathrm{OI_1} - \mathrm{OI_1}' = 0 \quad \cdots(\mathrm{d.6.20})$$

(d.6.20)より

$$\mathrm{OI_1} = \mathrm{OI_1}' \quad \cdots(\mathrm{d.6.21})$$

(d.6.1)(d.6.21)より

$$\mathrm{OI_1} = \frac{1}{2}(\mathrm{OE_1} \cdot \mathrm{OI_3} + \mathrm{OE_1} - 2) \quad \cdots(\mathrm{d.6.22})$$

(d.6.14)(d.6.21)より

$$\mathrm{OI_1} - \mathrm{OI_2}' = \mathrm{OI_1} - \mathrm{OI_2} \quad \cdots(\mathrm{d.6.23})$$

(d.6.23)より

$$\mathrm{OI_2} = \mathrm{OI_2}' \quad \cdots(\mathrm{d.6.24})$$

(d.6.2)(d.6.24)より

$$\mathrm{OI}_2 = \frac{1}{2}\left(\mathrm{OE}_1 \cdot \mathrm{OI}_4 - \mathrm{OE}_1 + 2\right) \quad \cdots(\mathrm{d.6.25})$$

(d.5.13)より、次の関係式を満たす点$\mathrm{I}_3{}'$をとることができる。

$$\mathrm{OI}_3{}' = \frac{1}{2}\left(\mathrm{OE}_2 \cdot \mathrm{OI}_2 + \mathrm{OE}_2 + 2\right) \quad \cdots(\mathrm{d.6.26})$$

(d.5.19)より、次の関係式を満たす点$\mathrm{I}_4{}'$をとることができる。

$$\mathrm{OI}_4{}' = \frac{1}{2}\left(\mathrm{OE}_2 \cdot \mathrm{OI}_1 - \mathrm{OE}_2 - 2\right) \quad \cdots(\mathrm{d.6.27})$$

(d.6.26)(d.6.27)より

$$\mathrm{OI}_3{}' - \mathrm{OI}_4{}' = \frac{1}{2}\left(\mathrm{OE}_2 \cdot \mathrm{OI}_2 + \mathrm{OE}_2 + 2\right) - \frac{1}{2}\left(\mathrm{OE}_2 \cdot \mathrm{OI}_1 - \mathrm{OE}_2 - 2\right) \quad \cdots(\mathrm{d.6.28})$$

(d.6.28)より

$$\mathrm{OI}_3{}' - \mathrm{OI}_4{}' = -\frac{1}{2}\mathrm{OE}_2\left(\mathrm{OI}_1 - \mathrm{OI}_2\right) + \mathrm{OE}_2 + 2 \quad \cdots(\mathrm{d.6.29})$$

(d.2.7)(d.6.29)より

$$\mathrm{OI}_3{}' - \mathrm{OI}_4{}' = -\frac{1}{2}\mathrm{OE}_2 \cdot \mathrm{OE}_1 + \mathrm{OE}_2 + 2 \quad \cdots(\mathrm{d.6.30})$$

(d.1.14)(d.6.30)より

$$\mathrm{OI}_3{}' - \mathrm{OI}_4{}' = -\frac{1}{2} \cdot 4 + \mathrm{OE}_2 + 2 \quad \cdots(\mathrm{d.6.31})$$

(d.6.31)より

$$\mathrm{OI}_3{}' - \mathrm{OI}_4{}' = \mathrm{OE}_2 \quad \cdots(\mathrm{d.6.32})$$

(d.6.26)(d.6.27)より

$$\mathrm{OI}_3{}' \cdot \mathrm{OI}_4{}' = \frac{1}{2}\left(\mathrm{OE}_2 \cdot \mathrm{OI}_2 + \mathrm{OE}_2 + 2\right) \cdot \frac{1}{2}\left(\mathrm{OE}_2 \cdot \mathrm{OI}_1 - \mathrm{OE}_2 - 2\right) \quad \cdots(\mathrm{d.6.33})$$

(d.6.33)より

$$\mathrm{OI}_3{}' \cdot \mathrm{OI}_4{}' = \frac{1}{4}\mathrm{OE}_2{}^2\mathrm{OI}_1 \cdot \mathrm{OI}_2 + \frac{1}{4}\mathrm{OE}_2\left(\mathrm{OE}_2 + 2\right)\left(\mathrm{OI}_1 - \mathrm{OI}_2\right) - \frac{1}{4}\left(\mathrm{OE}_2 + 2\right)^2$$

$\cdots(\mathrm{d.6.34})$

(d.2.7)(d.2.9)(d.6.34)より

$$\mathrm{OI}_3{}' \cdot \mathrm{OI}_4{}' = \frac{1}{4}\mathrm{OE}_2{}^2 \cdot 1 + \frac{1}{4}\mathrm{OE}_2\left(\mathrm{OE}_2 + 2\right)\mathrm{OE}_1 - \frac{1}{4}\left(\mathrm{OE}_2 + 2\right)^2 \quad \cdots(\mathrm{d.6.35})$$

(d.6.35)より

$$\mathrm{OI}_3{}' \cdot \mathrm{OI}_4{}' = \frac{1}{4}\mathrm{OE}_1 \cdot \mathrm{OE}_2\left(\mathrm{OE}_2 + 2\right) - \mathrm{OE}_2 - 1 \quad \cdots(\mathrm{d.6.36})$$

(d.1.14)(d.6.36)より

$$\mathrm{OI_3}' \cdot \mathrm{OI_4}' = \frac{1}{4} \cdot 4(\mathrm{OE_2} + 2) - \mathrm{OE_2} - 1 \quad \cdots(\mathrm{d.6.37})$$

(d.6.37)より

$$\mathrm{OI_3}' \cdot \mathrm{OI_4}' = 1 \quad \cdots(\mathrm{d.6.38})$$

(d.2.20)(d.6.32)より

$$\mathrm{OI_3}' - \mathrm{OI_4}' = \mathrm{OI_3} - \mathrm{OI_4} \quad \cdots(\mathrm{d.6.39})$$

(d.2.22)(d.6.38)より

$$\mathrm{OI_3}' \cdot \mathrm{OI_4}' = \mathrm{OI_3} \cdot \mathrm{OI_4} \quad \cdots(\mathrm{d.6.40})$$

計算により

$$\left(\mathrm{OI_3} - \mathrm{OI_3}'\right)\left(\mathrm{OI_3} + \mathrm{OI_4}'\right) = \mathrm{OI_3}^2 - \left(\mathrm{OI_3}' - \mathrm{OI_4}'\right)\mathrm{OI_3} - \mathrm{OI_3}' \cdot \mathrm{OI_4}' \quad \cdots(\mathrm{d.6.41})$$

(d.6.39)(d.6.40)(d.6.41)より

$$\left(\mathrm{OI_3} - \mathrm{OI_3}'\right)\left(\mathrm{OI_3} + \mathrm{OI_4}'\right) = \mathrm{OI_3}^2 - (\mathrm{OI_3} - \mathrm{OI_4})\mathrm{OI_3} - \mathrm{OI_3} \cdot \mathrm{OI_4} \quad \cdots(\mathrm{d.6.42})$$

(d.6.42)より

$$\left(\mathrm{OI_3} - \mathrm{OI_3}'\right)\left(\mathrm{OI_3} + \mathrm{OI_4}'\right) = 0 \quad \cdots(\mathrm{d.6.43})$$

$\mathrm{OI_3} > 0$、$\mathrm{OI_4}' > 0$より

$$\mathrm{OI_3} + \mathrm{OI_4}' > 0 \quad \cdots(\mathrm{d.6.44})$$

(d.6.43)(d.6.44)より

$$\mathrm{OI_3} - \mathrm{OI_3}' = 0 \quad \cdots(\mathrm{d.6.45})$$

(d.6.45)より

$$\mathrm{OI_3} = \mathrm{OI_3}' \quad \cdots(\mathrm{d.6.46})$$

(d.6.26)(d.6.46)より

$$\mathrm{OI_3} = \frac{1}{2}(\mathrm{OE_2} \cdot \mathrm{OI_2} + \mathrm{OE_2} + 2) \quad \cdots(\mathrm{d.6.47})$$

(d.6.39)(d.6.46)より

$$\mathrm{OI_3} - \mathrm{OI_4}' = \mathrm{OI_3} - \mathrm{OI_4} \quad \cdots(\mathrm{d.6.48})$$

(d.6.48)より

$$\mathrm{OI}_4 = \mathrm{OI}_4{}' \quad \cdots(\mathrm{d}.6.49)$$

(d.6.27)(d.6.49)より

$$\mathrm{OI}_4 = \frac{1}{2}\left(\mathrm{OE}_2 \cdot \mathrm{OI}_1 - \mathrm{OE}_2 - 2\right) \quad \cdots(\mathrm{d}.6.50)$$

(d.2.32)より

$$\mathrm{OI}_2{}^2 + 2\mathrm{OI}_3 - 4 = \left(1 - \mathrm{OE}_1 \cdot \mathrm{OI}_2\right) + 2\mathrm{OI}_3 - 4 \quad \cdots(\mathrm{d}.6.51)$$

(d.6.51)より

$$\mathrm{OI}_2{}^2 + 2\mathrm{OI}_3 - 4 = -\mathrm{OE}_1 \cdot \mathrm{OI}_2 + 2\mathrm{OI}_3 - 3 \quad \cdots(\mathrm{d}.6.52)$$

(d.6.47)(d.6.52)より

$$\mathrm{OI}_2{}^2 + 2\mathrm{OI}_3 - 4 = -\mathrm{OE}_1 \cdot \mathrm{OI}_2 + \left(\mathrm{OE}_2 \cdot \mathrm{OI}_2 + \mathrm{OE}_2 + 2\right) - 3 \quad \cdots(\mathrm{d}.6.53)$$

(d.6.53)より

$$\mathrm{OI}_2{}^2 + 2\mathrm{OI}_3 - 4 = \mathrm{OI}_2\left(\mathrm{OE}_2 - \mathrm{OE}_1\right) + \mathrm{OE}_2 - 1 \quad \cdots(\mathrm{d}.6.54)$$

(d.1.12)(d.6.54)より

$$\mathrm{OI}_2{}^2 + 2\mathrm{OI}_3 - 4 = \mathrm{OI}_2 \cdot 1 + \left(\mathrm{OE}_1 + 1\right) - 1 \quad \cdots(\mathrm{d}.6.55)$$

(d.6.55)より

$$\mathrm{OI}_2{}^2 + 2\mathrm{OI}_3 - 4 = \mathrm{OI}_2 + \mathrm{OE}_1 \quad \cdots(\mathrm{d}.6.56)$$

(d.2.7)(d.6.56)より

$$\mathrm{OI}_2{}^2 + 2\mathrm{OI}_3 - 4 = \mathrm{OI}_1 \quad \cdots(\mathrm{d}.6.57)$$

(d.6.57)より

$$\mathrm{OI}_1 = \mathrm{OI}_2{}^2 + 2\mathrm{OI}_3 - 4 \quad \cdots(\mathrm{d}.6.58)$$

(d.2.29)より

$$-\mathrm{OI}_1{}^2 + 2\mathrm{OI}_4 + 4 = -\left(\mathrm{OE}_1 \cdot \mathrm{OI}_1 + 1\right) + 2\mathrm{OI}_4 + 4 \quad \cdots(\mathrm{d}.6.59)$$

(d.6.59)より

$$-\mathrm{OI}_1{}^2 + 2\mathrm{OI}_4 + 4 = -\mathrm{OE}_1 \cdot \mathrm{OI}_1 + 2\mathrm{OI}_4 + 3 \quad \cdots(\mathrm{d}.6.60)$$

(d.6.50)(d.6.60)より

$$-\mathrm{OI}_1{}^2 + 2\mathrm{OI}_4 + 4 = -\mathrm{OE}_1 \cdot \mathrm{OI}_1 + \left(\mathrm{OE}_2 \cdot \mathrm{OI}_1 - \mathrm{OE}_2 - 2\right) + 3 \quad \cdots(\mathrm{d}.6.61)$$

(d.6.61)より

$$-\mathrm{OI}_1{}^2+2\mathrm{OI}_4+4=\mathrm{OI}_1\left(\mathrm{OE}_2-\mathrm{OE}_1\right)-\mathrm{OE}_2+1 \quad \cdots(\mathrm{d}.6.62)$$

(d.1.12)(d.6.62)より

$$-\mathrm{OI}_1{}^2+2\mathrm{OI}_4+4=\mathrm{OI}_1\cdot 1-\left(\mathrm{OE}_1+1\right)+1 \quad \cdots(\mathrm{d}.6.63)$$

(d.6.63)より

$$-\mathrm{OI}_1{}^2+2\mathrm{OI}_4+4=\mathrm{OI}_1-\mathrm{OE}_1 \quad \cdots(\mathrm{d}.6.64)$$

(d.2.7)(d.6.64)より

$$-\mathrm{OI}_1{}^2+2\mathrm{OI}_4+4=\mathrm{OI}_2 \quad \cdots(\mathrm{d}.6.65)$$

(d.6.65)より

$$\mathrm{OI}_2=-\mathrm{OI}_1{}^2+2\mathrm{OI}_4+4 \quad \cdots(\mathrm{d}.6.66)$$

(d.2.38)より

$$-\mathrm{OI}_4{}^2-2\mathrm{OI}_2+4=-\left(1-\mathrm{OE}_2\cdot\mathrm{OI}_4\right)-2\mathrm{OI}_2+4 \quad \cdots(\mathrm{d}.6.67)$$

(d.6.67)より

$$-\mathrm{OI}_4{}^2-2\mathrm{OI}_2+4=\mathrm{OE}_2\cdot\mathrm{OI}_4-2\mathrm{OI}_2+3 \quad \cdots(\mathrm{d}.6.68)$$

(d.6.25)(d.6.68)より

$$-\mathrm{OI}_4{}^2-2\mathrm{OI}_2+4=\mathrm{OE}_2\cdot\mathrm{OI}_4-\left(\mathrm{OE}_1\cdot\mathrm{OI}_4-\mathrm{OE}_1+2\right)+3 \quad \cdots(\mathrm{d}.6.69)$$

(d.6.69)より

$$-\mathrm{OI}_4{}^2-2\mathrm{OI}_2+4=\mathrm{OI}_4\left(\mathrm{OE}_2-\mathrm{OE}_1\right)+\mathrm{OE}_1+1 \quad \cdots(\mathrm{d}.6.70)$$

(d.1.12)(d.6.70)より

$$-\mathrm{OI}_4{}^2-2\mathrm{OI}_2+4=\mathrm{OI}_4\cdot 1+\left(\mathrm{OE}_2-1\right)+1 \quad \cdots(\mathrm{d}.6.71)$$

(d.6.71)より

$$-\mathrm{OI}_4{}^2-2\mathrm{OI}_2+4=\mathrm{OI}_4+\mathrm{OE}_2 \quad \cdots(\mathrm{d}.6.72)$$

(d.2.20)(d.6.72)より

$$-\mathrm{OI}_4{}^2-2\mathrm{OI}_2+4=\mathrm{OI}_3 \quad \cdots(\mathrm{d}.6.73)$$

(d.6.73)より

$$\mathrm{OI}_3=-\mathrm{OI}_4{}^2-2\mathrm{OI}_2+4 \quad \cdots(\mathrm{d}.6.74)$$

(d.2.35)より

$$OI_3{}^2 - 2OI_1 - 4 = (OE_2 \cdot OI_3 + 1) - 2OI_1 - 4 \quad \cdots(d.6.75)$$

(d.6.75)より

$$OI_3{}^2 - 2OI_1 - 4 = OE_2 \cdot OI_3 - 2OI_1 - 3 \quad \cdots(d.6.76)$$

(d.6.22)(d.6.76)より

$$OI_3{}^2 - 2OI_1 - 4 = OE_2 \cdot OI_3 - (OE_1 \cdot OI_3 + OE_1 - 2) - 3 \quad \cdots(d.6.77)$$

(d.6.77)より

$$OI_3{}^2 - 2OI_1 - 4 = OI_3 (OE_2 - OE_1) - OE_1 - 1 \quad \cdots(d.6.78)$$

(d.1.12)(d.6.78)より

$$OI_3{}^2 - 2OI_1 - 4 = OI_3 \cdot 1 - (OE_2 - 1) - 1 \quad \cdots(d.6.79)$$

(d.6.79)より

$$OI_3{}^2 - 2OI_1 - 4 = OI_3 - OE_2 \quad \cdots(d.6.80)$$

(d.2.20)(d.6.80)より

$$OI_3{}^2 - 2OI_1 - 4 = OI_4 \quad \cdots(d.6.81)$$

(d.6.81)より

$$OI_4 = OI_3{}^2 - 2OI_1 - 4 \quad \cdots(d.6.82)$$