（証明ⅳ）

計算により
$$4.12^2 = 16.9744 \,\text{、}\, 4.13^2 = 17.0569 \quad \cdots\text{(d.4.1)}$$

(d.4.1)より
$$4.12^2 < 17 < 4.13^2 \quad \cdots\text{(d.4.2)}$$

(d.1.18)(d.4.2)より
$$4.12^2 < \left(\mathrm{OE}_2 + \mathrm{OE}_1\right)^2 < 4.13^2 \quad \cdots\text{(d.4.3)}$$

$\mathrm{OE}_2 + \mathrm{OE}_1 > 0$ であるから、(d.4.3)より
$$4.12 < \mathrm{OE}_2 + \mathrm{OE}_1 < 4.13 \quad \cdots\text{(d.4.4)}$$

(d.1.12)(d.4.4)より
$$4.12 - 1 < \left(\mathrm{OE}_2 + \mathrm{OE}_1\right) - \left(\mathrm{OE}_2 - \mathrm{OE}_1\right) < 4.13 - 1 \quad \cdots\text{(d.4.5)}$$

(d.4.5)より
$$1.56 < \mathrm{OE}_1 < 1.565 \quad \cdots\text{(d.4.6)}$$

(d.4.6)より
$$1.56 < \mathrm{OE}_1 < 1.57 \quad \cdots\text{(d.4.7)}$$

(d.1.12)(d.4.4)より
$$4.12 + 1 < \left(\mathrm{OE}_2 + \mathrm{OE}_1\right) + \left(\mathrm{OE}_2 - \mathrm{OE}_1\right) < 4.13 + 1 \quad \cdots\text{(d.4.8)}$$

(d.4.8)より
$$2.56 < \mathrm{OE}_2 < 2.565 \quad \cdots\text{(d.4.9)}$$

(d.4.9)より
$$2.56 < \mathrm{OE}_2 < 2.57 \quad \cdots\text{(d.4.10)}$$

(d.2.7)(d.4.7)より
$$1.56 < \mathrm{OI}_1 - \mathrm{OI}_2 < 1.57 \quad \cdots\text{(d.4.11)}$$

(d.4.7)より
$$6.4336 < \mathrm{OE}_1^{\,2} + 4 < 6.4649 \quad \cdots\text{(d.4.12)}$$

(d.2.13)(d.4.12)より
$$6.4336 < \left(\mathrm{OI}_1 + \mathrm{OI}_2\right)^2 < 6.4649 \quad \cdots\text{(d.4.13)}$$

計算により
$$2.53^2 = 6.4009 \,\text{、}\, 2.55^2 = 6.5025 \quad \cdots\text{(d.4.14)}$$

(d.4.13)(d.4.14)より
$$2.53^2 < \left(\mathrm{OI}_1 + \mathrm{OI}_2\right)^2 < 2.55^2 \quad \cdots\text{(d.4.15)}$$

$\mathrm{OI}_1 + \mathrm{OI}_2 > 0$ であるから、(d.4.15)より
$$2.53 < \mathrm{OI}_1 + \mathrm{OI}_2 < 2.55 \quad \cdots\text{(d.4.16)}$$

(d.4.11)(d.4.16)より
$$2.53 + 1.56 < \left(\mathrm{OI}_1 + \mathrm{OI}_2\right) + \left(\mathrm{OI}_1 - \mathrm{OI}_2\right) < 2.55 + 1.57 \quad \cdots\text{(d.4.17)}$$

(d.4.17)より

$$2.045 < OI_1 < 2.06 \quad \cdots (d.4.18)$$

(d.4.18)より

$$2.04 < OI_1 < 2.06 \quad \cdots (d.4.19)$$

(d.4.11)(d.4.16)より

$$2.53 - 1.57 < \left(OI_1 + OI_2 \right) - \left(OI_1 - OI_2 \right) < 2.55 - 1.56 \quad \cdots (d.4.20)$$

(d.4.20)より

$$0.48 < OI_2 < 0.495 \quad \cdots (d.4.21)$$

(d.4.21)より

$$0.48 < OI_2 < 0.5 \quad \cdots (d.4.22)$$

(d.2.20)(d.4.10)より

$$2.56 < OI_3 - OI_4 < 2.57 \quad \cdots (d.4.23)$$

(d.4.10)より

$$10.5336 < OE_2{}^2 + 4 < 10.6049 \quad \cdots (d.4.24)$$

計算により

$$3.24^2 = 10.4976 、 3.26^2 = 10.6276 \quad \cdots (d.4.25)$$

(d.4.24)(d.4.25)より

$$3.24^2 < OE_2{}^2 + 4 < 3.26^2 \quad \cdots (d.4.26)$$

(d.2.26)(d.4.26)より

$$3.24^2 < \left(OI_3 + OI_4 \right)^2 < 3.26^2 \quad \cdots (d.4.27)$$

$OI_3 + OI_4 > 0$ であるから、(d.4.27)より

$$3.24 < OI_3 + OI_4 < 3.26 \quad \cdots (d.4.28)$$

(d.4.24)(d.4.28)より

$$3.24 + 2.56 < \left(OI_3 + OI_4 \right) + \left(OI_3 - OI_4 \right) < 3.26 + 2.57 \quad \cdots (d.4.29)$$

(d.4.29)より

$$2.9 < OI_3 < 2.915 \quad \cdots (d.4.30)$$

(d.4.30)より

$$2.9 < OI_3 < 2.92 \quad \cdots (d.4.31)$$

(d.4.24)(d.4.28)より

$$3.24 - 2.57 < \left(OI_3 + OI_4 \right) - \left(OI_3 - OI_4 \right) < 3.26 - 2.56 \quad \cdots (d.4.32)$$

(d.4.32)より

$$0.335 < OI_4 < 0.35 \quad \cdots (d.4.33)$$

(d.4.33)より

$$0.33 < OI_4 < 0.35 \quad \cdots (d.4.34)$$

(d.3.8)(d.4.19)より

$$2.04 < OM_1 + OM_2 < 2.06 \quad \cdots (d.4.35)$$

(d.4.19)(d.4.34)より

$$2.04^2 - 4 \times 0.35 < OI_1{}^2 - 4OI_4 < 2.06^2 - 4 \times 0.33 \quad \cdots(d.4.36)$$

(d.4.36)より

$$2.7616 < OI_1{}^2 - 4OI_4 < 2.9236 \quad \cdots(d.4.37)$$

(d.3.14)(d.4.37)より

$$2.7616 < \left(OM_1 - OM_2\right)^2 < 2.9236 \quad \cdots(d.4.38)$$

計算により

$$1.66^2 = 2.7556 \text{ 、 } 1.71^2 = 2.9241 \quad \cdots(d.4.39)$$

(d.4.38)(d.4.39)より

$$1.66^2 < \left(OM_1 - OM_2\right)^2 < 1.71^2 \quad \cdots(d.4.40)$$

仮定より

$$OM_1 > OM_2 \quad \cdots(d.4.41)$$

(d.4.41)より

$$OM_1 - OM_2 > 0 \quad \cdots(d.4.42)$$

(d.4.40)(d.4.42)より

$$1.66 < OM_1 - OM_2 < 1.71 \quad \cdots(d.4.43)$$

(d.4.35)(d.4.43)より

$$2.04 + 1.66 < \left(OM_1 + OM_2\right) + \left(OM_1 - OM_2\right) < 2.06 + 1.71 \quad \cdots(d.4.44)$$

(d.4.44)より

$$1.85 < OM_1 < 1.885 \quad \cdots(d.4.45)$$

(d.4.45)より

$$1.85 < OM_1 < 1.89 \quad \cdots(d.4.46)$$

(d.4.35)(d.4.43)より

$$2.04 - 1.71 < \left(OM_1 + OM_2\right) - \left(OM_1 - OM_2\right) < 2.06 - 1.66 \quad \cdots(d.4.47)$$

(d.4.47)より

$$0.165 < OM_2 < 0.2 \quad \cdots(d.4.48)$$

(d.4.49)より

$$0.16 < OM_2 < 0.2 \quad \cdots(d.4.49)$$

(d.3.22)(d.4.22)より

$$0.48 < OM_4 - OM_3 < 0.5 \quad \cdots(d.4.50)$$

(d.4.22)(d.4.31)より

$$0.48^2 + 4 \times 2.9 < OI_2{}^2 + 4OI_3 < 0.5^2 + 4 \times 2.92 \quad \cdots(d.4.51)$$

(d.4.51)より

$$11.8304 < \mathrm{OI_2}^2 + 4\mathrm{OI_3} < 11.93 \quad \cdots\text{(d.4.52)}$$

計算により

$$3.43^2 = 11.7649 \text{、} \quad 3.46^2 < 11.9716 \quad \cdots\text{(d.4.53)}$$

(d.4.52)(d.4.53)より

$$3.43^2 < \mathrm{OI_2}^2 + 4\mathrm{OI_3} < 3.46^2 \quad \cdots\text{(d.4.54)}$$

(d.3.28)(d.4.54)より

$$3.43^2 < \left(\mathrm{OM_4} + \mathrm{OM_3}\right)^2 < 3.46^2 \quad \cdots\text{(d.4.55)}$$

$\mathrm{OM_4} + \mathrm{OM_3} > 0$ であるから、(d.4.55)より

$$3.43 < \mathrm{OM_4} + \mathrm{OM_3} < 3.46 \quad \cdots\text{(d.4.56)}$$

(d.4.50)(d.4.56)より

$$3.43 - 0.5 < \left(\mathrm{OM_4} + \mathrm{OM_3}\right) - \left(\mathrm{OM_4} - \mathrm{OM_3}\right) < 3.46 - 0.48 \quad \cdots\text{(d.4.57)}$$

(d.4.57)より

$$1.465 < \mathrm{OM_3} < 1.49 \quad \cdots\text{(d.4.58)}$$

(d.4.58)より

$$1.46 < \mathrm{OM_3} < 1.49 \quad \cdots\text{(d.4.59)}$$

(d.4.50)(d.4.56)より

$$3.43 + 0.48 < \left(\mathrm{OM_4} + \mathrm{OM_3}\right) + \left(\mathrm{OM_4} - \mathrm{OM_3}\right) < 3.46 + 0.5 \quad \cdots\text{(d.4.60)}$$

(d.4.60)より

$$1.955 < \mathrm{OM_4} < 1.98 \quad \cdots\text{(d.4.61)}$$

(d.4.61)より

$$1.95 < \mathrm{OM_4} < 1.98 \quad \cdots\text{(d.4.62)}$$