（証明ⅱ）

仮定より

$$OF_1 = \frac{1}{2}OE_1 \quad \cdots(d.2.1)$$

$$F_1I_1 = F_1I_2 \quad \cdots(d.2.2)$$

O、F_1、I_1はこの順に同一直線上にあるから

$$F_1I_1 = OI_1 - OF_1 \quad \cdots(d.2.3)$$

F_1、O、I_2はこの順に同一直線上にあるから

$$F_1I_2 = OF_1 + OI_2 \quad \cdots(d.2.4)$$

(d.2.2)(d.2.3)(d.2.4)より

$$OI_1 - OF_1 = OF_1 + OI_2 \quad \cdots(d.2.5)$$

(d.2.5)より

$$OI_1 - OI_2 = 2OF_1 \quad \cdots(d.2.6)$$

(d.2.1)(d.2.6)より

$$OI_1 - OI_2 = OE_1 \quad \cdots(d.2.7)$$

点Z_4は円F_1上にあるから、方べきの定理より

$$OI_1 \cdot OI_2 = OZ_2 \cdot OZ_4 \quad \cdots(d.2.8)$$

(d.1.1)(d.2.8)より

$$OI_1 \cdot OI_2 = 1 \quad \cdots(d.2.9)$$

(d.2.7)より

$$(OI_1 - OI_2)^2 = OE_1{}^2 \quad \cdots(d.2.10)$$

(d.2.10)より

$$(OI_1 + OI_2)^2 - 4OI_1 \cdot OI_2 = OE_1{}^2 \quad \cdots(d.2.11)$$

(d.2.9)(d.2.11)より

$$(OI_1 + OI_2)^2 - 4 = OE_1{}^2 \quad \cdots(d.2.12)$$

(d.2.12)より

$$(OI_1 + OI_2)^2 = OE_1{}^2 + 4 \quad \cdots(d.2.13)$$

仮定より

$$OF_2 = \frac{1}{2}OE_2 \quad \cdots(d.2.14)$$

$$F_2I_3 = F_2I_4 \quad \cdots(d.2.15)$$

O、F_2、I_3はこの順に同一直線上にあるから

$$F_2I_3 = OI_3 - OF_2 \quad \cdots(d.2.16)$$

F_2、O、I_4はこの順に同一直線上にあるから

$$F_2I_4 = OF_2 + OI_4 \quad \cdots(d.2.17)$$

(d.2.15)(d.2.16)(d.2.17)より

$$\mathrm{OI_3} - \mathrm{OF_2} = \mathrm{OF_2} + \mathrm{OI_4} \quad \cdots\text{(d.2.18)}$$

(d.2.18)より

$$\mathrm{OI_3} - \mathrm{OI_4} = 2\mathrm{OF_2} \quad \cdots\text{(d.2.19)}$$

(d.2.14)(d.2.19)より

$$\mathrm{OI_3} - \mathrm{OI_4} = \mathrm{OE_2} \quad \cdots\text{(d.2.20)}$$

点 $\mathrm{Z_4}$ は円 $\mathrm{F_2}$ 上にあるから、方べきの定理より

$$\mathrm{OI_3} \cdot \mathrm{OI_4} = \mathrm{OZ_2} \cdot \mathrm{OZ_4} \quad \cdots\text{(d.2.21)}$$

(d.1.1)(d.2.21)より

$$\mathrm{OI_3} \cdot \mathrm{OI_4} = 1 \quad \cdots\text{(d.2.22)}$$

(d.2.20)より

$$\left(\mathrm{OI_3} - \mathrm{OI_4}\right)^2 = \mathrm{OE_2}^2 \quad \cdots\text{(d.2.23)}$$

(d.2.23)より

$$\left(\mathrm{OI_3} + \mathrm{OI_4}\right)^2 - 4\mathrm{OI_3} \cdot \mathrm{OI_4} = \mathrm{OE_2}^2 \quad \cdots\text{(d.2.24)}$$

(d.2.22)(d.2.24)より

$$\left(\mathrm{OI_3} + \mathrm{OI_4}\right)^2 - 4 = \mathrm{OE_2}^2 \quad \cdots\text{(d.2.25)}$$

(d.2.25)より

$$\left(\mathrm{OI_3} + \mathrm{OI_4}\right)^2 = \mathrm{OE_2}^2 + 4 \quad \cdots\text{(d.2.26)}$$

(d.2.7)より

$$\mathrm{OI_1}^2 - \mathrm{OI_1} \cdot \mathrm{OI_2} = \mathrm{OE_1} \cdot \mathrm{OI_1} \quad \cdots\text{(d.2.27)}$$

(d.2.9)(d.2.27)より

$$\mathrm{OI_1}^2 - 1 = \mathrm{OE_1} \cdot \mathrm{OI_1} \quad \cdots\text{(d.2.28)}$$

(d.2.28)より

$$\mathrm{OI_1}^2 = \mathrm{OE_1} \cdot \mathrm{OI_1} + 1 \quad \cdots\text{(d.2.29)}$$

(d.2.7)より

$$\mathrm{OI_1} \cdot \mathrm{OI_2} - \mathrm{OI_2}^2 = \mathrm{OE_1} \cdot \mathrm{OI_2} \quad \cdots\text{(d.2.30)}$$

(d.2.9)(d.2.30)より

$$1 - \mathrm{OI_2}^2 = \mathrm{OE_1} \cdot \mathrm{OI_2} \quad \cdots\text{(d.2.31)}$$

(d.2.31)より

$$\mathrm{OI_2}^2 = 1 - \mathrm{OE_1} \cdot \mathrm{OI_2} \quad \cdots\text{(d.2.32)}$$

(d.2.20)より

$$\mathrm{OI_3}^2 - \mathrm{OI_3} \cdot \mathrm{OI_4} = \mathrm{OE_2} \cdot \mathrm{OI_3} \quad \cdots\text{(d.2.33)}$$

(d.2.22)(d.2.33)より

$$\mathrm{OI_3}^2 - 1 = \mathrm{OE_2} \cdot \mathrm{OI_3} \quad \cdots\text{(d.2.34)}$$

(d.2.34)より

$$\mathrm{OI_3}^2 = \mathrm{OE_2} \cdot \mathrm{OI_3} + 1 \quad \cdots\text{(d.2.35)}$$

(d.2.20)より

$$\mathrm{OI_3} \cdot \mathrm{OI_4} - \mathrm{OI_4}^2 = \mathrm{OE_2} \cdot \mathrm{OI_4} \quad \cdots\text{(d.2.36)}$$

(d.2.22)(d.2.36)より

$$1 - \mathrm{OI_4}^2 = \mathrm{OE_2} \cdot \mathrm{OI_4} \quad \cdots\text{(d.2.37)}$$

(d.2.37)より

$$\mathrm{OI_4}^2 = 1 - \mathrm{OE_2} \cdot \mathrm{OI_4} \quad \cdots\text{(d.2.38)}$$