（証明ｉ）

仮定より

$$\mathrm{OZ}_1 = \mathrm{OZ}_2 = \mathrm{OZ}_3 = \mathrm{OZ}_4 = 1 \quad \cdots(\mathrm{d}.1.1)$$

$$\mathrm{Z}_1\mathrm{Z}_3 \perp \mathrm{Z}_2\mathrm{Z}_4 \quad \cdots(\mathrm{d}.1.2)$$

$$\mathrm{OB}_1 = \frac{1}{2} \quad \cdots(\mathrm{d}.1.3)$$

$$\mathrm{OC}_1 = 4 \quad \cdots(\mathrm{d}.1.4)$$

$$\mathrm{B}_1\mathrm{D}_1 \perp \mathrm{OZ}_1 \quad \cdots(\mathrm{d}.1.5)$$

$$\mathrm{D}_1\mathrm{E}_1 = \mathrm{D}_1\mathrm{E}_2 \quad \cdots(\mathrm{d}.1.6)$$

(d.1.5)(d.1.6)より

$$\mathrm{B}_1\mathrm{E}_1 = \mathrm{B}_1\mathrm{E}_2 \quad \cdots(\mathrm{d}.1.7)$$

B_1、O、E_1はこの順に同一直線上にあるから、

$$\mathrm{B}_1\mathrm{E}_1 = \mathrm{OB}_1 + \mathrm{OE}_1 \quad \cdots(\mathrm{d}.1.8)$$

O、B_1、E_2はこの順に同一直線上にあるから、

$$\mathrm{B}_1\mathrm{E}_2 = \mathrm{OE}_2 - \mathrm{OB}_1 \quad \cdots(\mathrm{d}.1.9)$$

(d.1.7)(d.1.8)(d.1.9)より

$$\mathrm{OB}_1 + \mathrm{OE}_1 = \mathrm{OE}_2 - \mathrm{OB}_1 \quad \cdots(\mathrm{d}.1.10)$$

(d.1.10)より

$$\mathrm{OE}_2 - \mathrm{OE}_1 = 2\mathrm{OB}_1 \quad \cdots(\mathrm{d}.1.11)$$

(d.1.3)(d.1.11)より

$$\mathrm{OE}_2 - \mathrm{OE}_1 = 1 \quad \cdots(\mathrm{d}.1.12)$$

点C_1は円D_1上にあるから、方べきの定理より

$$\mathrm{OE}_1 \cdot \mathrm{OE}_2 = \mathrm{OZ}_2 \cdot \mathrm{OC}_1 \quad \cdots(\mathrm{d}.1.13)$$

(d.1.1)(d.1.4)(d.1.13)より

$$\mathrm{OE}_1 \cdot \mathrm{OE}_2 = 4 \quad \cdots(\mathrm{d}.1.14)$$

(d.1.12)より

$$\left(\mathrm{OE}_2 - \mathrm{OE}_1\right)^2 = 1 \quad \cdots(\mathrm{d}.1.15)$$

(d.1.15)より

$$\left(\mathrm{OE}_2 + \mathrm{OE}_1\right)^2 - 4\mathrm{OE}_1 \cdot \mathrm{OE}_2 = 1 \quad \cdots(\mathrm{d}.1.16)$$

(d.1.14)(d.1.16)より

$$\left(\mathrm{OE}_2 + \mathrm{OE}_1\right)^2 - 16 = 1 \quad \cdots(\mathrm{d}.1.17)$$

(d.1.17)より

$$\left(\mathrm{OE}_2 + \mathrm{OE}_1\right)^2 = 17 \quad \cdots(\mathrm{d}.1.18)$$

(d.1.12)より

$$\mathrm{OE_1} \cdot \mathrm{OE_2} - \mathrm{OE_1}^2 = \mathrm{OE_1} \quad \cdots\text{(d.1.19)}$$

(d.1.14)(d.1.19)より

$$4 - \mathrm{OE_1}^2 = \mathrm{OE_1} \quad \cdots\text{(d.1.20)}$$

(d.1.20)より

$$\mathrm{OE_1}^2 = 4 - \mathrm{OE_1} \quad \cdots\text{(d.1.21)}$$

(d.1.12)より

$$\mathrm{OE_2}^2 - \mathrm{OE_1} \cdot \mathrm{OE_2} = \mathrm{OE_2} \quad \cdots\text{(d.1.22)}$$

(d.1.14)(d.1.22)より

$$\mathrm{OE_2}^2 - 4 = \mathrm{OE_2} \quad \cdots\text{(d.1.23)}$$

(d.1.23)より

$$\mathrm{OE_2}^2 = 4 + \mathrm{OE_2} \quad \cdots\text{(d.1.24)}$$